

TExES | Texas Examinations of Educator Standards

Preparation Manual



115 Mathematics 4-8

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Chapter 1

Introduction to the Mathematics 4–8 Test and Suggestions for Using this Test Preparation Manual

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OVERVIEW

The State Board for Educator Certification (SBEC) has approved Texas educator standards that delineate what the beginning educator should know and be able to do. These standards, which are based on the state-required curriculum for students — the Texas Essential Knowledge and Skills (TEKS) — form the basis for the Texas Examinations of Educator Standards® (TExES®) program. This initiative, directed by Texas Education Agency (TEA), will affect all areas of Texas education — from the more than 100 approved Texas Educator Preparation Programs (EPPs) to the more than 7,000 Texas school campuses. This standards-based system reflects TEA's commitment to help align Texas education from kindergarten through college. TEA's role in this K–16 initiative will ensure that newly certified Texas educators have the essential knowledge and skills to teach the TEKS to the state's public school students.

This manual is designed to help examinees prepare for the TEXES test in this field. Its purpose is to familiarize examinees with the competencies to be tested, test question formats and pertinent study resources. EPP staff may also find this information useful as they help examinees prepare for careers as Texas educators.

KEY FEATURES OF THE MANUAL

- *List of competencies that will be tested*
- Strategies for answering multiple-choice questions
- Sample test questions and answer key

If you have any questions after reading this preparation manual or you would like additional information about the TEXES tests or the educator standards, please visit the SBEC website at **www.sbec.state.tx.us**.

USING THE TEST FRAMEWORK

The Texas Examinations of Educator Standards (TExES) tests measure the content knowledge required of an entry-level educator in a particular field in Texas public schools. This manual is designed to guide your preparation by helping you become familiar with the material to be covered on the test you are planning to take, identify areas where you feel you may be weak and increase your knowledge in those areas by helping you design a study plan.

When preparing for this test, you should focus on the competencies and descriptive statements, which delineate the content that is eligible for testing. A portion of the content is represented in the sample questions that are included in this manual. These test questions represent only a sampling of questions. Thus, your test preparation should focus on the competencies and descriptive statements and not simply on the sample questions.

ORGANIZATION OF THE TEXES TEST FRAMEWORK

The test framework is based on the educator standards for this field.

The content covered by this test is organized into broad areas of content called domains. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of competencies. Each competency is composed of two major parts:

- 1. the **competency statement**, which broadly defines what an entry-level educator in this field in Texas public schools should know and be able to do, and
- 2. the **descriptive statements**, which describe in greater detail the knowledge and skills eligible for testing.

The educator standards being assessed within each domain are listed for reference at the beginning of the test framework, which begins on page 12. These are followed by a complete set of the framework's competencies and descriptive statements.

An example of a competency and its accompanying descriptive statements is provided below.

SAMPLE COMPETENCY

Mathematics 4–8

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COMPETENCY 001
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THE TEACHER UNDERSTANDS THE STRUCTURE OF NUMBER SYSTEMS, THE DEVELOPMENT OF A SENSE OF QUANTITY AND THE RELATIONSHIP BETWEEN QUANTITY AND SYMBOLIC REPRESENTATIONS.

SAMPLE DESCRIPTIVE STATEMENTS

- A. Analyzes the structure of numeration systems and the roles of place value and zero in the base ten system.
- B. Understands the relative magnitude of whole numbers, integers, rational numbers and real numbers.
- C. Demonstrates an understanding of a variety of models for representing numbers (e.g., fraction strips, diagrams, patterns, shaded regions, number lines).
- D. Demonstrates an understanding of equivalency among different representations of rational numbers.
- E. Selects appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- F. Understands the characteristics of the set of whole numbers, integers, rational numbers, real numbers and complex numbers (e.g., commutativity, order, closure, identity elements, inverse elements, density).
- G. Demonstrates an understanding of how some situations that have no solution in one number system (e.g., whole numbers, integers, rational numbers) have solutions in another number system (e.g., real numbers, complex numbers).

STUDYING FOR THE TEXES TEST

The following steps may be helpful in preparing for the TExES test.

- 1. Identify the information the test will cover by reading through the test competencies (see Chapter 3). Within each domain of this TExES test, each competency will receive approximately equal coverage.
- 2. Read each competency with its descriptive statements in order to get a more specific idea of the knowledge you will be required to demonstrate on the test. You may wish to use this review of the competencies to set priorities for your study time.
- 3. Review the "Preparation Resources" section of this guide (Appendix B) for possible resources to consult. Also, compile key materials from your preparation coursework that are aligned with the competencies.
- 4. Study this manual for approaches to taking the TExES test.
- 5. When using resources, concentrate on the key skills and important abilities that are discussed in the competencies and descriptive statements.
- 6. Use the study plan document (Appendix A of this guide) to help you plan your study.

NOTE: This preparation manual is the only TEXES test study material endorsed by Texas Education Agency (TEA) for this field. Other preparation materials may not accurately reflect the content of the test or the policies and procedures of the TEXES program.

Chapter 2

Background Information on the TExES Testing Program

THE TEXES TESTS FOR TEXAS TEACHERS

As required by the Texas Education Code §21.048, successful performance on educator certification examinations is required for the issuance of a Texas educator certificate. Each TExES test is a criterion-referenced examination designed to measure the knowledge and skills delineated in the corresponding TExES test framework. Each test framework is based on standards that were developed by Texas educators and other education stakeholders.

Each TEXES test is designed to measure the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess. The tests include both individual (stand-alone) test questions and questions that are arranged in clustered sets based on real-world situations faced by educators.

DEVELOPMENT OF THE NEW TEXES TESTS

Committees of Texas educators and members of the community guide the development of the new TExES tests by participating in each stage of the test development process. These working committees are composed of Texas educators from public and charter schools, university and EPP faculty, education service center staff, representatives from professional educator organizations, content experts and members of the business community. The committees are balanced in terms of position, affiliation, years of experience, ethnicity, gender and geographical location. The committee membership is rotated during the development process so that numerous Texas stakeholders may be actively involved. The steps in the process to develop the TExES tests are described below.

- 1. **Develop Standards.** Committees are established to recommend what the beginning educator should know and be able to do. Using the Texas Essential Knowledge and Skills (TEKS) as the focal point, draft standards are prepared to define the knowledge and skills required of the beginning educator.
- 2. **Review Standards.** Committees review and revise the draft standards. The revised draft standards are then placed on the State Board for Educator Certification (SBEC) website for public review and comment. These comments are used to prepare a final draft of the standards that will be presented to the SBEC Board for discussion, the State Board of Education (SBOE) for review and comment and the SBEC Board for approval. Standards not based specifically on the TEKS, such as those for librarians and counselors, are proposed as rule by the SBEC Board; sent to the SBOE for its 90-day review; and, if not rejected by the SBOE, adopted by the SBEC Board.
- 3. **Develop Test Frameworks.** Committees review and revise draft test frameworks that are based on the standards. These frameworks outline the specific competencies to be measured on the new TExES tests. Draft frameworks are not finalized until after the standards are approved and the job analysis/content validation survey (see #4) is complete.

- 4. **Conduct Job Analysis/Content Validation Surveys.** A representative sample of Texas educators who practice in or prepare individuals for each of the fields for which an educator certificate has been proposed are surveyed to determine the relative job importance of each competency outlined in the test framework for that content area. Frameworks are revised as needed following an analysis of the survey responses.
- 5. **Develop and Review New Test Questions.** The test contractor develops draft questions that are designed to measure the competencies described in the test framework. Committees review the newly developed test questions that have been written to reflect the competencies in the new test frameworks. Committee members scrutinize the draft questions for appropriateness of content and difficulty; clarity; match to the competencies; and potential ethnic, gender and regional bias.
- 6. **Conduct Pilot Test of New Test Questions.** All of the newly developed test questions that have been deemed acceptable by the question review committees are then administered to an appropriate sample of candidates for certification.
- 7. **Review Pilot Test Data.** Pilot test results are reviewed to ensure that the test questions are valid, reliable and free from bias.
- 8. Administer TExES Tests. New TExES tests are constructed to reflect the competencies, and the tests are administered to candidates for certification.
- 9. Set Passing Standard. A Standard Setting Committee convenes to review performance data from the initial administration of each new TExES test and to recommend a final passing standard for that test. The SBEC Board considers this recommendation as it establishes a passing score on the test.

TAKING THE TEXES TEST AND RECEIVING SCORES

Please refer to the current TEXES *Registration Bulletin* or the ETS TEXES website at **www.texes.ets.org** for information on test dates, test centers, fees, registration procedures and program policies.

Your score report will be available to you in your testing account on the ETS TEXES online registration system by 5 p.m. Central time on the score reporting date indicated in the *Registration Bulletin*. The report will indicate whether you have passed the test and will include:

- A total test scaled score. Scaled scores are reported to allow for the comparison of scores on the same content-area test taken on different test administration dates. The total scaled score is not the percentage of questions answered correctly and is not determined by averaging the number of questions answered correctly in each domain.
 - For all TEXES tests, the score scale is 100–300 with a scaled score of 240 as the minimum passing score. This score represents the minimum level of competency required to be an entry-level educator in this field in Texas public schools.
- Your performance in the major content domains of the test and in the specific content competencies of the test.
 - This information may be useful in identifying strengths and weaknesses in your content preparation and can be used for further study or for preparing to retake the test. However, it is important to use caution when interpreting scores reported by domain and competency as these scores are typically based on a smaller number of items than the total score and therefore may not be as reliable as the total score.
- A link to information that will help you understand the score scale and interpret your results.

A score report will not be available to you if you are absent or choose to cancel your score.

For more information about scores or to access scores online, go to www.texes.ets.org.

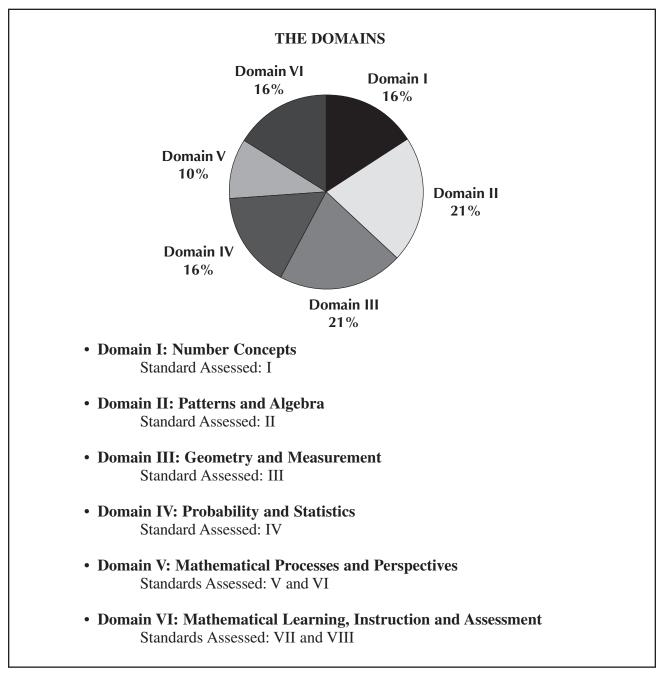
EDUCATOR STANDARDS

Complete, approved educator standards are posted on the SBEC website at www.sbec.state.tx.us.

Chapter 3

Study Topics





TEST FRAMEWORK FOR FIELD 115: MATHEMATICS 4-8

TOTAL TEST BREAKDOWN

- Exam is offered as a paper-based or computer-administered test
- 90 Multiple-Choice Questions (80 Scored Questions*)

*Your final scaled score will be based only on scored questions.

THE STANDARDS

DOMAIN I — NUMBER CONCEPTS (approximately 16% of the test)

MATHEMATICS STANDARD I:

Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

DOMAIN II — PATTERNS AND ALGEBRA (approximately 21% of the test)

MATHEMATICS STANDARD II:

Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

DOMAIN III — GEOMETRY AND MEASUREMENT (approximately 21% of the test)

MATHEMATICS STANDARD III:

Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

DOMAIN IV — PROBABILITY AND STATISTICS (approximately 16% of the test)

MATHEMATICS STANDARD IV:

Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) in order to prepare students to use mathematics.

DOMAIN V — MATHEMATICAL PROCESSES AND PERSPECTIVES (approximately 10% of the test)

MATHEMATICS STANDARD V:

Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics and to communicate mathematically.

MATHEMATICS STANDARD VI:

Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the interrelationship between society and mathematics, the structure of mathematics and the evolving nature of mathematics and mathematical knowledge.

DOMAIN VI — MATHEMATICAL LEARNING, INSTRUCTION AND ASSESSMENT (approximately 16% of the test)

MATHEMATICS STANDARD VII:

Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures and concepts, knows typical errors students make, and uses this knowledge to plan, organize and implement instruction; to meet curriculum goals; and to teach all students to understand and use mathematics.

MATHEMATICS STANDARD VIII:

Mathematical Assessment: The mathematics teacher understands assessment and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

COMPETENCIES

DOMAIN I — NUMBER CONCEPTS

COMPETENCY 001

THE TEACHER UNDERSTANDS THE STRUCTURE OF NUMBER SYSTEMS, THE DEVELOPMENT OF A SENSE OF QUANTITY AND THE RELATIONSHIP BETWEEN QUANTITY AND SYMBOLIC REPRESENTATIONS.

- A. Analyzes the structure of numeration systems and the roles of place value and zero in the base ten system.
- B. Understands the relative magnitude of whole numbers, integers, rational numbers and real numbers.
- C. Demonstrates an understanding of a variety of models for representing numbers (e.g., fraction strips, diagrams, patterns, shaded regions, number lines).
- D. Demonstrates an understanding of equivalency among different representations of rational numbers.
- E. Selects appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- F. Understands the characteristics of the set of whole numbers, integers, rational numbers, real numbers and complex numbers (e.g., commutativity, order, closure, identity elements, inverse elements, density).
- G. Demonstrates an understanding of how some situations that have no solution in one number system (e.g., whole numbers, integers, rational numbers) have solutions in another number system (e.g., real numbers, complex numbers).

COMPETENCY 002

THE TEACHER UNDERSTANDS NUMBER OPERATIONS AND COMPUTATIONAL ALGORITHMS.

The beginning teacher:

- A. Works proficiently with real and complex numbers and their operations.
- B. Analyzes and describes relationships between number properties, operations and algorithms for the four basic operations involving integers, rational numbers and real numbers.
- C. Uses a variety of concrete and visual representations to demonstrate the connections between operations and algorithms.
- D. Justifies procedures used in algorithms for the four basic operations with integers, rational numbers and real numbers and analyzes error patterns that may occur in their application.
- E. Relates operations and algorithms involving numbers to algebraic procedures (e.g., adding fractions to adding rational expressions, division of integers to division of polynomials).
- F. Extends and generalizes the operations on rationals and integers to include exponents, their properties and their applications to the real numbers.

COMPETENCY 003

THE TEACHER UNDERSTANDS IDEAS OF NUMBER THEORY AND USES NUMBERS TO MODEL AND SOLVE PROBLEMS WITHIN AND OUTSIDE OF MATHEMATICS.

- A. Demonstrates an understanding of ideas from number theory (e.g., prime factorization, greatest common divisor) as they apply to whole numbers, integers and rational numbers and uses these ideas in problem situations.
- B. Uses integers, rational numbers and real numbers to describe and quantify phenomena such as money, length, area, volume and density.
- C. Applies knowledge of place value and other number properties to develop techniques of mental mathematics and computational estimation.
- D. Applies knowledge of counting techniques such as permutations and combinations to quantify situations and solve problems.
- E. Applies properties of the real numbers to solve a variety of theoretical and applied problems.

DOMAIN II — PATTERNS AND ALGEBRA

COMPETENCY 004

THE TEACHER UNDERSTANDS AND USES MATHEMATICAL REASONING TO IDENTIFY, EXTEND AND ANALYZE PATTERNS AND UNDERSTANDS THE RELATIONSHIPS AMONG VARIABLES, EXPRESSIONS, EQUATIONS, INEQUALITIES, RELATIONS AND FUNCTIONS.

The beginning teacher:

- A. Uses inductive reasoning to identify, extend and create patterns using concrete models, figures, numbers and algebraic expressions.
- B. Formulates implicit and explicit rules to describe and construct sequences verbally, numerically, graphically and symbolically.
- C. Makes, tests, validates and uses conjectures about patterns and relationships in data presented in tables, sequences or graphs.
- D. Gives appropriate justification of the manipulation of algebraic expressions.
- E. Illustrates the concept of a function using concrete models, tables, graphs and symbolic and verbal representations.
- F. Uses transformations to illustrate properties of functions and relations and to solve problems.

COMPETENCY 005

THE TEACHER UNDERSTANDS AND USES LINEAR FUNCTIONS TO MODEL AND SOLVE PROBLEMS.

- A. Demonstrates an understanding of the concept of linear function using concrete models, tables, graphs and symbolic and verbal representations.
- B. Demonstrates an understanding of the connections among linear functions, proportions and direct variation.
- C. Determines the linear function that best models a set of data.
- D. Analyzes the relationship between a linear equation and its graph.
- E. Uses linear functions, inequalities and systems to model problems.
- F. Uses a variety of representations and methods (e.g., numerical methods, tables, graphs, algebraic techniques) to solve systems of linear equations and inequalities.
- G. Demonstrates an understanding of the characteristics of linear models and the advantages and disadvantages of using a linear model in a given situation.

COMPETENCY 006

THE TEACHER UNDERSTANDS AND USES NONLINEAR FUNCTIONS AND RELATIONS TO MODEL AND SOLVE PROBLEMS.

The beginning teacher:

- A. Uses a variety of methods to investigate the roots (real and complex), vertex and symmetry of a quadratic function or relation.
- B. Demonstrates an understanding of the connections among geometric, graphic, numeric and symbolic representations of quadratic functions.
- C. Analyzes data and represents and solves problems involving exponential growth and decay.
- D. Demonstrates an understanding of the connections among proportions, inverse variation and rational functions.
- E. Understands the effects of transformations such as $f(x \pm c)$ on the graph of a nonlinear function f(x).
- F. Applies properties, graphs and applications of nonlinear functions to analyze, model and solve problems.
- G. Uses a variety of representations and methods (e.g., numerical methods, tables, graphs, algebraic techniques) to solve systems of quadratic equations and inequalities.
- H. Understands how to use properties, graphs and applications of nonlinear relations including polynomial, rational, radical, absolute value, exponential, logarithmic, trigonometric and piecewise functions and relations to analyze, model and solve problems.

COMPETENCY 007

THE TEACHER USES AND UNDERSTANDS THE CONCEPTUAL FOUNDATIONS OF CALCULUS RELATED TO TOPICS IN MIDDLE SCHOOL MATHEMATICS.

- A. Relates topics in middle school mathematics to the concept of limit in sequences and series.
- B. Relates the concept of average rate of change to the slope of the secant line and instantaneous rate of change to the slope of the tangent line.
- C. Relates topics in middle school mathematics to the area under a curve.
- D. Demonstrates an understanding of the use of calculus concepts to answer questions about rates of change, areas, volumes and properties of functions and their graphs.

DOMAIN III — GEOMETRY AND MEASUREMENT

COMPETENCY 008

THE TEACHER UNDERSTANDS MEASUREMENT AS A PROCESS.

The beginning teacher:

- A. Selects and uses appropriate units of measurement (e.g., temperature, money, mass, weight, area, capacity, density, percents, speed, acceleration) to quantify, compare and communicate information.
- B. Develops, justifies and uses conversions within measurement systems.
- C. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- D. Describes the precision of measurement and the effects of error on measurement.
- E. Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.

COMPETENCY 009

THE TEACHER UNDERSTANDS THE GEOMETRIC RELATIONSHIPS AND AXIOMATIC STRUCTURE OF EUCLIDEAN GEOMETRY.

The beginning teacher:

- A. Understands concepts and properties of points, lines, planes, angles, lengths and distances.
- B. Analyzes and applies the properties of parallel and perpendicular lines.
- C. Uses the properties of congruent triangles to explore geometric relationships and prove theorems.
- D. Describes and justifies geometric constructions made using a compass and straight edge and other appropriate technologies.
- E. Applies knowledge of the axiomatic structure of Euclidean geometry to justify and prove theorems.

COMPETENCY 010

THE TEACHER ANALYZES THE PROPERTIES OF TWO- AND THREE-DIMENSIONAL FIGURES.

- A. Uses and understands the development of formulas to find lengths, perimeters, areas and volumes of basic geometric figures.
- B. Applies relationships among similar figures, scale and proportion and analyzes how changes in scale affect area and volume measurements.

- C. Uses a variety of representations (e.g., numeric, verbal, graphic, symbolic) to analyze and solve problems involving two- and three-dimensional figures such as circles, triangles, polygons, cylinders, prisms and spheres.
- D. Analyzes the relationship among three-dimensional figures and related twodimensional representations (e.g., projections, cross-sections, nets) and uses these representations to solve problems.

COMPETENCY 011

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THE TEACHER UNDERSTANDS TRANSFORMATIONAL GEOMETRY AND RELATES ALGEBRA TO GEOMETRY AND TRIGONOMETRY USING THE CARTESIAN COORDINATE SYSTEM.
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The beginning teacher:

- A. Describes and justifies geometric constructions made using a reflection device and other appropriate technologies.
- B. Uses translations, reflections, glide-reflections and rotations to demonstrate congruence and to explore the symmetries of figures.
- C. Uses dilations (expansions and contractions) to illustrate similar figures and proportionality.
- D. Uses symmetry to describe tessellations and shows how they can be used to illustrate geometric concepts, properties and relationships.
- E. Applies concepts and properties of slope, midpoint, parallelism and distance in the coordinate plane to explore properties of geometric figures and solve problems.
- F. Applies transformations in the coordinate plane.
- G. Uses the unit circle in the coordinate plane to explore properties of trigonometric functions.

DOMAIN IV - PROBABILITY AND STATISTICS

COMPETENCY 012

THE TEACHER UNDERSTANDS HOW TO USE GRAPHICAL AND NUMERICAL TECHNIQUES TO EXPLORE DATA, CHARACTERIZE PATTERNS AND DESCRIBE DEPARTURES FROM PATTERNS.

- A. Organizes and displays data in a variety of formats (e.g., tables, frequency distributions, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- B. Applies concepts of center, spread, shape and skewness to describe a data distribution.
- C. Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

- D. Demonstrates an understanding of measures of central tendency (e.g., mean, median, mode) and dispersion (e.g., range, interquartile range, variance, standard deviation).
- E. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.
- F. Calculates and interprets percentiles and quartiles.

COMPETENCY 013

THE TEACHER UNDERSTANDS THE THEORY OF PROBABILITY.

The beginning teacher:

- A. Explores concepts of probability through data collection, experiments and simulations.
- B. Uses the concepts and principles of probability to describe the outcome of simple and compound events.
- C. Generates, simulates and uses probability models to represent a situation.
- D. Determines probabilities by constructing sample spaces to model situations.
- E. Solves a variety of probability problems using combinations, permutations and geometric probability (i.e., probability as the ratio of two areas).
- F. Uses the binomial, geometric and normal distributions to solve problems.

COMPETENCY 014

THE TEACHER UNDERSTANDS THE RELATIONSHIP AMONG PROBABILITY THEORY, SAMPLING AND STATISTICAL INFERENCE AND HOW STATISTICAL INFERENCE IS USED IN MAKING AND EVALUATING PREDICTIONS.

- A. Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.
- B. Demonstrates an understanding of random samples, sample statistics and the relationship between sample size and confidence intervals.
- C. Applies knowledge of the use of probability to make observations and draw conclusions from single variable data and to describe the level of confidence in the conclusion.
- D. Makes inferences about a population using binomial, normal and geometric distributions.
- E. Demonstrates an understanding of the use of techniques such as scatter plots, regression lines, correlation coefficients and residual analysis to explore bivariate data and to make and evaluate predictions.

DOMAIN V — MATHEMATICAL PROCESSES AND PERSPECTIVES

COMPETENCY 015

THE TEACHER UNDERSTANDS MATHEMATICAL REASONING AND PROBLEM SOLVING.

The beginning teacher:

- A. Demonstrates an understanding of proof, including indirect proof, in mathematics.
- B. Applies correct mathematical reasoning to derive valid conclusions from a set of premises.
- C. Demonstrates an understanding of the use of inductive reasoning to make conjectures and deductive methods to evaluate the validity of conjectures.
- D. Applies knowledge of the use of formal and informal reasoning to explore, investigate and justify mathematical ideas.
- E. Recognizes that a mathematical problem can be solved in a variety of ways and selects an appropriate strategy for a given problem.
- F. Evaluates the reasonableness of a solution to a given problem.
- G. Applies content knowledge to develop a mathematical model of a real-world situation and analyzes and evaluates how well the model represents the situation.
- H. Demonstrates an understanding of estimation and evaluates its appropriate uses.

COMPETENCY 016

THE TEACHER UNDERSTANDS MATHEMATICAL CONNECTIONS WITHIN AND OUTSIDE OF MATHEMATICS AND HOW TO COMMUNICATE MATHEMATICAL IDEAS AND CONCEPTS.

- A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of circle as a quadratic function in *r*, probability as the ratio of two areas).
- B. Uses mathematics to model and solve problems in other disciplines, such as art, music, science, social science and business.
- C. Expresses mathematical statements using developmentally appropriate language, standard English, mathematical language and symbolic mathematics.
- D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphic, pictorial, symbolic, concrete).
- E. Demonstrates an understanding of the use of visual media such as graphs, tables, diagrams and animations to communicate mathematical information.
- F. Uses the language of mathematics as a precise means of expressing mathematical ideas.
- G. Understands the structural properties common to the mathematical disciplines.

DOMAIN VI -- MATHEMATICAL LEARNING, INSTRUCTION AND ASSESSMENT

COMPETENCY 017

THE TEACHER UNDERSTANDS HOW CHILDREN LEARN AND DEVELOP MATHEMATICAL SKILLS, PROCEDURES AND CONCEPTS.

The beginning teacher:

- A. Applies theories and principles of learning mathematics to plan appropriate instructional activities for all students.
- B. Understands how students differ in their approaches to learning mathematics with regard to diversity.
- C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.
- D. Understands how learning may be assisted through the use of mathematics manipulatives and technological tools.
- E. Understands how to motivate students and actively engage them in the learning process by using a variety of interesting, challenging and worthwhile mathematical tasks in individual, small-group and large-group settings.
- F. Understands how to provide instruction along a continuum from concrete to abstract.
- G. Recognizes the implications of current trends and research in mathematics and mathematics education.

COMPETENCY 018

THE TEACHER UNDERSTANDS HOW TO PLAN, ORGANIZE AND IMPLEMENT INSTRUCTION USING KNOWLEDGE OF STUDENTS, SUBJECT MATTER AND STATEWIDE CURRICULUM (TEXAS ESSENTIAL KNOWLEDGE AND SKILLS [TEKS]) TO TEACH ALL STUDENTS TO USE MATHEMATICS.

- A. Demonstrates an understanding of a variety of instructional methods, tools and tasks that promote students' ability to do mathematics described in the TEKS.
- B. Understands planning strategies for developing mathematical instruction as a discipline of interconnected concepts and procedures.
- C. Develops clear learning goals to plan, deliver, assess and reevaluate instruction based on the TEKS.
- D. Understands procedures for developing instruction that establishes transitions between concrete, symbolic and abstract representations of mathematical knowledge.

- E. Applies knowledge of a variety of instructional delivery methods, such as individual, structured small-group and large-group formats.
- F. Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop and improve mathematical skills and procedures.
- G. Demonstrates an understanding of a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.
- H. Understands how technological tools and manipulatives can be used appropriately to assist students in developing, comprehending and applying mathematical concepts.
- I. Understands how to relate mathematics to students' lives and a variety of careers and professions.

COMPETENCY 019

THE TEACHER UNDERSTANDS ASSESSMENT AND USES A VARIETY OF FORMAL AND INFORMAL ASSESSMENT TECHNIQUES TO MONITOR AND GUIDE MATHEMATICS INSTRUCTION AND TO EVALUATE STUDENT PROGRESS.

- A. Demonstrates an understanding of the purpose, characteristics and uses of various assessments in mathematics, including formative and summative assessments.
- B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.
- C. Demonstrates an understanding of how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.
- D. Understands how to evaluate a variety of assessment methods and materials for reliability, validity, absence of bias, clarity of language and appropriateness of mathematical level.
- E. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning for all students, including English-language learners.

Chapter 4

Succeeding on Multiple-Choice Questions

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APPROACHES TO ANSWERING MULTIPLE-CHOICE QUESTIONS

The purpose of this section is to describe multiple-choice question formats that you will see on the Mathematics 4–8 test and to suggest possible ways to approach thinking about and answering the multiple-choice questions. However, these approaches are not intended to replace familiar test-taking strategies with which you are already comfortable and that work for you.

The Mathematics 4–8 test is designed to include a total of 90 multiple-choice questions, out of which 80 are scored. Your final scaled score will be based only on scored questions. The questions that are not scored are being pilot tested in order to collect information about how these questions will perform under actual testing conditions. These questions are not identified on the test.

All multiple-choice questions on this test are designed to assess your knowledge of the content described in the test framework. In most cases, you are expected to demonstrate more than just your ability to recall factual information. You may be asked to solve a multi-step problem; analyze and interpret mathematical information in a variety of formats; determine a mathematical function that models a given situation; or supply information needed to prove a mathematical statement.

When you are ready to respond to a multiple-choice question, you must choose one of four answer options labeled A, B, C and D. Leave no questions unanswered. Nothing is subtracted from your score if you answer a question incorrectly. Questions for which you mark no answer or more than one answer are not counted in scoring. Your score will be determined by the number of questions for which you select the best answer.

Calculators. Scientific calculators will be provided at the test center. See the TExES *Registration Bulletin* for the brand and model of the calculator that will be available.

Definitions and Formulas. A set of definitions and formulas will be provided. A copy of those definitions and formulas is also provided in Chapter 5 of this preparation manual.

QUESTION FORMATS

You may see the following types of multiple-choice questions on the test.

- Single Questions
- Questions with Stimulus Material
- Clustered Questions

On the following pages, you will find descriptions of these commonly used question formats, along with suggested approaches for responding to each type of question. In the actual testing situation, if you are taking the paper-based version of the test, you may mark the test questions and/or write in the margins of your test booklet. **Your final response must be indicated on the answer sheet provided.** If you are taking the test via computer, you may write on the scratch paper provided at the testing center. **Your final response must be selected on the computer.**

SINGLE QUESTIONS

In the single-question format, a problem is presented as a direct question or an incomplete statement, and four answer options appear below the question. The following question is an example of this type. It tests knowledge of Mathematics 4–8 Competency 010: *The teacher analyzes the properties of two- and three-dimensional figures*.

EXAMPLE

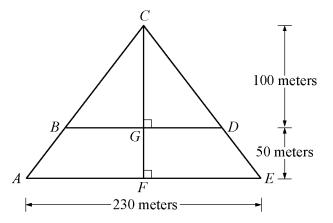
The Great Pyramid at Giza is approximately 150 meters high and has a square base approximately 230 meters on a side. What is the approximate area of a horizontal cross section of the pyramid taken 50 meters above its base?

- A. 5,880 square meters
- B. 11,760 square meters
- C. 23,510 square meters
- D. 35,270 square meters

SUGGESTED APPROACH

Read the question carefully and critically. Think about what it is asking and the situation it is describing. Eliminate any obviously wrong answers, select the correct answer choice and mark your answer.

The horizontal cross section will be a square in the plane parallel to the base of the pyramid and 50 meters above it. In order to estimate the area of the cross section, you will need to know the approximate length of one of its sides. This can be calculated using your knowledge of proportions and the properties of similar geometric figures. In solving problems that involve geometry, drawing a diagram is often helpful.



The figure shows a vertical cross section through the center of the square base of the pyramid perpendicular to a side of the base. The measurements given in the test question have been transferred to the diagram. Notice that since CG + GF = 150, and it is given that GF = 50, then CG = 100.

You must find *BD*, the length of the sides of the square cross section. Also note that $\triangle CBD$ and $\triangle CAE$ are similar because they have two angles whose measures are equal; they share $\angle C$ and the measure of $\angle B$ is equal to the measure of $\angle A$ since they are corresponding angles formed by a transversal and two parallel lines. Because the two triangles are similar, their altitudes and sides

must be proportional and you can write: $\frac{CG}{CF} = \frac{BD}{AE}$. Now substitute the values for the lengths of the

line segments to get $\frac{100}{150} = \frac{BD}{230}$. Solving this gives BD = 153.33. Since the horizontal cross section is a

square, its area is the square of the length of *BD*, or $(153.33)^2 = 23,511.11$ square feet. Now look at the response options. **The correct response is option C**, rounded to the nearest ten square meters.

Setting up the proportion incorrectly as $\frac{50}{150} = \frac{BD}{230}$ and using this value for the side of the cross

section leads to option A. Option B results from assuming that the cross section is an isosceles right triangle instead of a square, and option D comes from assuming that the area of the cross section is

 $\frac{100}{150} = \frac{2}{3}$ of the area of the base of the pyramid.

QUESTIONS WITH STIMULUS MATERIAL

Some questions on this test are preceded by stimulus material that relates to the question. Some types of stimulus material included on the test are geometric diagrams, charts, data tables, graphs, equations, samples of student work and descriptions of classroom situations. In such cases, you will generally be given information followed by an event to analyze, a problem to solve or a decision to make.

You can use several different approaches to respond to these types of questions. Some commonly used strategies are listed below.

Strategy 1	Skim the stimulus material to understand its purpose, its arrangement and/or its content. Then read the question and refer again to the stimulus material to obtain the specific information you need to answer the question.
Strategy 2	Read the question <i>before</i> considering the stimulus material. The theory behind this strategy is that the content of the question will help you identify the purpose of the stimulus material and locate the information you need to answer the question.
Strategy 3	Use a combination of both strategies; apply the "read the stimulus first" strategy with shorter, more familiar stimuli and the "read the question first" strategy with longer, more complex or less familiar stimuli. You can experiment with the sample questions in this manual and then use the strategy with which you are most comfortable when you take the actual test.

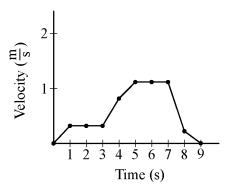
Whether you read the stimulus before or after you read the question, you should read it carefully and critically. If you are taking a paper-based test, you may want to underline its important points to help you answer the question.

As you consider questions set in educational contexts, try to enter into the identified teacher's frame of mind and use that teacher's point of view to answer the questions that accompany the stimulus. Be sure to consider the questions in terms of only the information provided in the stimulus — not in terms of your own class experiences or individual students you may have known.

EXAMPLE

First read the stimulus (a description of the concept being studied).

Use the diagram and the information below to answer the two questions that follow.



Students in a math class are investigating concepts related to motion in one dimension. The velocity-versus-time graph shows the velocity of a student walking in a straight line, collected at one-second intervals over a period of nine seconds.

Now you are prepared to address the first of the two questions associated with this stimulus. The first question measures Competency 007: *The teacher uses and understands the conceptual foundations of calculus related to topics in middle school mathematics*.

- 1. Which of the following methods could be used to estimate the student's acceleration between t = 3 and t = 5 seconds?
 - A. Find the average of the velocities at t = 3 and t = 5 seconds
 - B. Find the equation of the curve that best fits the data and evaluate it at t = 4 seconds
 - C. Find the length of the line connecting the velocities between t = 3 and t = 5 seconds
 - D. Find the slope of the line connecting the velocities at t = 3 and t = 5 seconds

SUGGESTED APPROACH

You are asked to estimate the acceleration of the student between 3 and 5 seconds, that is, the average acceleration over this time period. Average acceleration is the rate of change of velocity

with respect to time, $\frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$. Therefore, divide the difference in the velocities at 5 and 3 seconds

by the total time elapsed, here 5 - 3 = 2 seconds. You should recognize this expression as representing the slope of a line connecting two points, or the difference in the *y*-coordinates divided by the difference in the *x*-coordinates. Therefore, **option D is correct.**

Option A finds the average velocity in the time interval, while option B finds an expression for velocity as a function of time and interpolates how fast the student is moving at t = 4s. Option C determines the length of the curve and has no physical significance.

Now you are ready to answer the next question. The second question also measures Competency 007: *The teacher uses and understands the conceptual foundations of calculus related to topics in middle school mathematics.*

- 2. Which of the following methods could be used to estimate the total distance the student has traveled between t = 0 and t = 5 seconds?
 - A. Find the median value of the velocities from t = 0 and t = 5 seconds, inclusive.
 - B. Find the ratio of the velocities at t = 0 and t = 5 seconds.
 - C. Find the area under the curve between t = 0 and t = 5 seconds.
 - D. Find the average value of the velocity-over-time ratios for t = 0 and t = 5 seconds.

SUGGESTED APPROACH

In order to calculate the distance traveled by the student during a particular time interval, multiply the rate of travel by the length of time the student is moving; in other words, d = rt where d represents distance, r represents rate (velocity), and t represents time. For example, during the interval from t = 1 to t = 2 seconds, multiply the average velocity during the interval, approximately $0.25 \frac{m}{s}$, by the length of the interval, 2 - 1 = 1 second. This can be represented geometrically by the area of the rectangle of height $= 0.25 \frac{m}{s}$ and base = 1s under the curve between t = 1s and t = 2s. To

get an estimate of the total distance traveled by the student, you need to sum the distance traveled during each of the one-second intervals from 0 through 5 seconds. This is approximately equal to the area under the curve from t = 0 to t = 5 seconds. Therefore, **option C is correct**.

Option A gives the median value for the velocity, which by itself cannot be used to estimate the distance traveled by the student. The ratio of the velocities (option B) is not helpful in determining the total distance covered. Finding the average of the velocities at t = 0 and t = 5 (option D) is not by itself sufficient for calculating the distance traveled between those times.

CLUSTERED QUESTIONS

You may have one or more questions related to a single stimulus. When you have at least two questions related to a single stimulus, the group of questions is called a cluster.

Chapter 5

Multiple-Choice Practice Questions

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SAMPLE MULTIPLE-CHOICE QUESTIONS

This section presents some sample test questions for you to review as part of your preparation for the test. To demonstrate how each competency may be assessed, each sample question is accompanied by the competency that it measures. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements will not appear on the actual test.

An answer key follows the sample questions. The answer key lists the question number and correct answer for each sample test question. Please note that the answer key also lists the competency assessed by each question and that the sample questions are not necessarily presented in competency order.

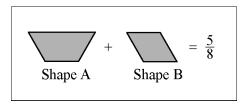
The sample questions are included to illustrate the formats and types of questions you will see on the test; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual test.

MULTIPLE-CHOICE PRACTICE QUESTIONS

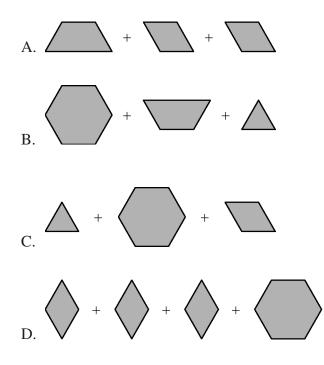
Definitions and Formulas for Mathematics 4-8			
CALCULUS	ALGEBRA		
First Derivative: $f'(x) = \frac{dy}{dx}$	i $i^2 = -1$ A^{-1} inverse of matrix A		
Second Derivative: $f''(x) = \frac{d^2y}{dx^2}$	$A = P\left(1 + \frac{r}{n}\right)^{nt}$ Compound interest, where A is the final value		
PROBABILITY P(A or B) = P(A) + P(B) - P(A and B)	P is the principal r is the interest rate t is the term n is the number of		
P(A and B) = P(A)P(B A) = P(B)P(A B)	divisions within the term		
	[x] = n Greatest integer function, where <i>n</i> is the integer such that $n \le x < n + 1$		
GEOMETRY	VOLUME		
Congruent Angles	Cylinder: (area of base) × height		
	Cone: $\frac{1}{3}$ (area of base) × height		
	Sphere: $\frac{4}{3}\pi r^3$		
Congruent Sides	Prism: (area of base) × height		
+ " +	AREA		
	Triangle: $\frac{1}{2}$ (base × height)		
Parallel Sides	Rhombus: $\frac{1}{2}$ (diagonal ₁ × diagonal ₂)		
11	Trapezoid: $\frac{1}{2}$ height (base ₁ + base ₂)		
Circumference of a Circle	Sphere: $4\pi r^2$		
$C = 2\pi r$	Circle: πr^2		
	Lateral surface area of cylinder: $2\pi rh$		
TRIGON Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	OMETRY		
	b a		
$c^{2} = a^{2} + b^{2} - 2ab \cos C$ Law of Cosines: $b^{2} = a^{2} + c^{2} - 2ac \cos B$ $a^{2} = b^{2} + c^{2} - 2bc \cos A$	$A \xrightarrow{c} B$		
END OF DEFINITIO	NS AND FORMULAS		

5

1. A fifth-grade class is using pattern blocks in the shape of congruent equilateral triangles to devise and solve problems involving fractions. One group devises the problem illustrated below.



Given that the sum of Shapes A and B represents $\frac{5}{8}$, which of the following represents $1\frac{1}{4}$?



2. Use the information below to answer the question that follows.

The Sahara Desert covers about 8.3×10^{13} square feet.

The average depth of the sand in the Sahara Desert is 200 feet.

A grain of sand has a volume of approximately 1.3×10^{-9} cubic feet.

Which of the following is the best estimate of the number of grains of sand in the Sahara Desert?

- A. 10²²
- B. 10²³
- C. 10²⁴
- D. 10²⁵

COMPETENCY 002

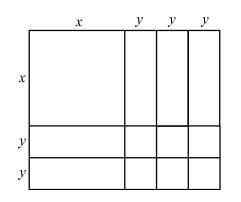
3. Use the addition problem below to answer the question that follows.

12	
29	
88	
+ 11	

When given the addition problem above, a student quickly said "140." When asked how she solved the problem, the student replied, "I added 88 and 12 to get 100, and 29 and 11 to get 40. Then I added these two numbers together." Which of the following two properties of addition did the student use in solving this problem?

- A. Associative and commutative
- B. Associative and additive identity
- C. Commutative and additive identity
- D. Distributive and additive inverse

4. Use the diagram below to answer the question that follows.



Which of the following expressions is represented by the areas of the rectangles in the diagram above?

- A. $xy^3 + xy^2$
- B. $x^2 + 5xy + 6y^2$
- C. $2x + y^3 + y^2$
- D. 2x + 5xy + 6y

COMPETENCY 003

5. Use the theorem below to answer the question that follows.

If the sum of a number's digits is divisible by three, the number is divisible by three.

Which of the following ways of expressing a three digit number, n, with hundreds digit a, tens digit b and units digit c, best demonstrates why this theorem is true?

A.
$$3n = 3a(100) + 3b(10) + 3c = 300a + 30b + 3c$$

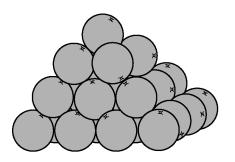
B.
$$n = a(100) + b(10) + c$$

- C. n = a(99+1) + b(9+1) + c = 99a + 9b + a + b + c
- D. n = a(70+30) + b(7+3) + c = 70a + 7b + 30a + 3b + c

- 6. The density of gold is 19.3 grams per cubic centimeter. What is the mass of a cube made of gold that measures 1.2 cm on a side?
 - A. 0.09 gram
 - B. 1.73 gram
 - C. 23.16 grams
 - D. 33.35 grams

COMPETENCY 004

7. Use the diagram below to answer the question that follows.



In a store display, grapefruit are stacked 4 levels high in the shape of a pyramid with a square base. Which of the following expressions can be used to determine how many grapefruit can be stacked in a pyramid *n* layers high?

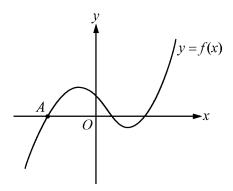
A.
$$n! = (1)(2)(3)\cdots(n)$$

B. $n^2 = 1 + 3 + 5 + \dots + (2n - 1)$

C.
$$\frac{n(n+1)}{2} = 1 + 2 + 3 + \dots + n$$

D.
$$\frac{n(n+1)(2n+1)}{6} = 1 + 4 + 9 + \dots + n^2$$

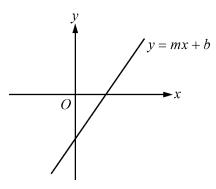
8. Use the graph below to answer the question that follows.



Which of the following equations could be solved to find the *x*-coordinate of point *A* on the graph of the function y = f(x) shown in the *xy*-plane?

- A. f(x) = 0
- B. f(0) = x
- C. f'(x) = 0
- D. f''(x) = 0

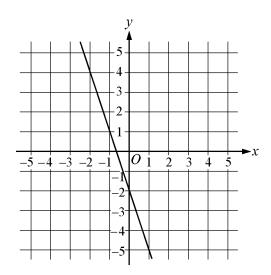
9. Use the graph below to answer the question that follows.



The graph represents an equation of the form y = mx + b. Which of the following statements about *m* and *b* are true?

- A. m > 0 and b > 0
- B. m > 0 and b < 0
- C. m < 0 and b > 0
- D. m < 0 and b < 0

10. Use the graph below to answer the question that follows.



The graph in the *xy*-plane of which of the following lines passes through the point (6, 12) and has the same slope as the line shown?

- A. y 2x = 0
- B. y + 3x = 30
- C. y 3x = -6
- D. y + 3x = 42

- 11. The function *f* is given by the equation $f(x) = x^2 + 6x + 7$ for all numbers *x*. Which of the following statements is true?
 - A. The graph of y = f(x) in the *xy*-plane has a vertex at the point (-3, -2)
 - B. The graph of y = f(x) in the *xy*-plane crosses the *x*-axis at two points, (-7,0) and (-6,0)
 - C. The function f does not have any real roots
 - D. As $x \to \infty$, $f(x) \to -\infty$

COMPETENCY 006

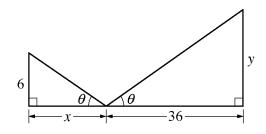
12. Use the function below to answer the question that follows.

$$f(x) = \frac{4}{x^2 - cx + 9}$$

When the rational function f given by the equation above is graphed in the xy-plane, what value of c will produce exactly one vertical asymptote?

- A. 0
- B. 4
- C. 6
- D. 9

13. Use the diagram below to answer the question that follows.



Which of the following statements describes how the value of y depends on the value of x in the triangles shown above?

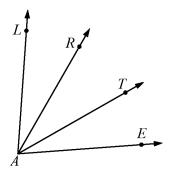
- A. *y* is directly proportional to x
- B. *y* is directly proportional to the square of x
- C. *y* is inversely proportional to x
- D. *y* is inversely proportional to the square of x

- 14. The shape of a city park is a square with sides that are each 1.5 miles long. What is the area of the park in acres? (1 square mile = 640 acres)
 - A. 720
 - B. 1280
 - C. 1440
 - D. 1920

- 15. Using a protractor, a student measures the interior angles in a triangle and finds the sum of the angles to be 176°. What is the percent error of this measurement?
 - A. 0.04%
 - B. 2.22%
 - C. 2.27%
 - D. 4.00%

COMPETENCY 009

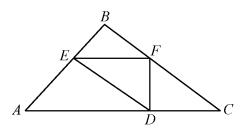
16. Use the diagram below to answer the question that follows.



In this diagram, $m \angle LAT = m \angle RAE$. Which property is most likely to be used in proving that $m \angle LAR = m \angle TAE$?

- A. The multiplication property of equality
- B. The addition property of equality
- C. The distributive property
- D. The properties of complementary angles

17. Use the figure below to answer the question that follows.



Note: Figure not drawn to scale.

In triangle *ABC* shown, *EF* is parallel to *AC*, and *ED* is parallel to *BC*. Which of the following statements gives sufficient additional information to conclude that triangles *AED*, *EBF*, *DFC* and *FED* are congruent?

- A. *BF* is equal to *FC*
- B. AED is a right angle
- C. *ABC* is a right triangle
- D. ABC is an isosceles triangle

18. Use the diagrams below to answer the question that follows.



The diagrams show three regular polygons and their central angles. Which of the following gives the measure of the central angle of a regular polygon, $\theta(n)$, as a function of the number of sides *n*, where *n* is greater than 2?

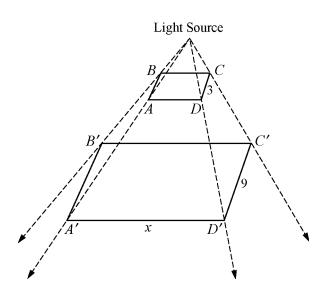
A.
$$\theta(n) = \frac{360}{n}$$

B. $\theta(n) = 360 - \frac{360}{n}$

C.
$$\theta(n) = 360 - n$$

D.
$$\theta(n) = 360 - n^2$$

19. Use the diagram below to answer the question that follows.



An overhead light source projects parallelogram ABCD to A'B'C'D'. Given that the projection is a dilation, which of the following expressions represents the perimeter of the smaller figure, ABCD, in terms of x?

A. 2x - 6

B. $3 + \frac{x}{3}$

C. 3*x* – 18

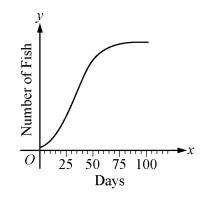
D. $6 + \frac{2}{3}x$

COMPETENCY OII

- 20. Which of the following is the reflection of the point $\left(\frac{3}{5}, \frac{4}{5}\right)$ through the line y = -x in the *xy* plane?
 - A. $\left(\frac{3}{5}, -\frac{4}{5}\right)$ B. $\left(-\frac{3}{5}, \frac{4}{5}\right)$ C. $\left(\frac{4}{5}, \frac{3}{5}\right)$ D. $\left(-\frac{4}{5}, -\frac{3}{5}\right)$

COMPETENCY 012

21. Use the graph below to answer the question that follows.



The function $f(x) = \frac{200}{0.25 + 6e^{-0.1x}}$ is used to model the number of fish in a certain

aquarium, where x is the number of days after the aquarium has been stocked with 32 fish. The graph of y = f(x) is shown in the xy-plane above. Which of the following statements is true about the number of fish in the aquarium?

- A. It approaches 0 as x increases
- B. It approaches 200 as x increases
- C. It approaches 800 as *x* increases
- D. It increases without bound as *x* increases

22. The amount of money, after taxes, spent by a family on various expenses during one month is given in the table below.

Type of Expense	Amount Spent in One Month		
Rent	\$750		
Food	\$575		
Utilities	\$120		
Car loan, gas and repairs	\$450		
Medical expenses	\$ 65		
Entertainment	\$120		
Credit card payment	\$350		
Miscellaneous expenses	\$95		
Total expenses	\$2525		

If the family constructs a pie chart using these figures, what is the approximate measure of the central angle of the sector used to represent the percentage of total expenses spent on food?

A. 23°

B. 63°

C. 77°

D. 82°

23. Two teams meet in a playoff series at the end of the regular season. Team A won 55 of 81 games played in its home stadium during the regular season, while Team B won 48 of 81 games played in its home stadium. The first two games of the series will be played in Team A's home stadium, the next two games in Team B's home stadium. In the absence of any other information, which expression is equal to the probability that Team A will win the first four games in a row?

 $A. \left(\frac{55}{81}\right) \left(\frac{55}{81}\right) \left(\frac{55}{81}\right) \left(\frac{55}{81}\right)$ $B. \left(\frac{55}{81}\right) \left(\frac{55}{81}\right) \left(\frac{48}{81}\right) \left(\frac{48}{81}\right)$ $C. \left(\frac{55}{81}\right) \left(\frac{55}{81}\right) \left(\frac{33}{81}\right) \left(\frac{33}{81}\right)$ $D. \left(\frac{55}{81}\right) \left(\frac{55}{81}\right) \left(\frac{26}{81}\right) \left(\frac{26}{81}\right)$

COMPETENCY 013

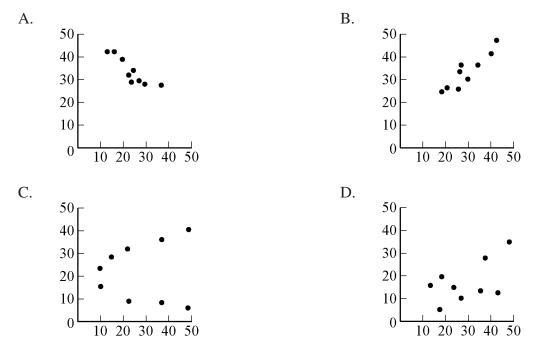
24. Use the table below to answer the question that follows.

	Number of Piec Specifie		
Cuts of Beef	High Fat Content	Low Fat Content	Total
Flank Steaks	74	386	460
Rump Roasts	258	142	400
Total	332	528	860

A United States Department of Agriculture (USDA) inspector graded cuts of beef at a meat packing plant. If a piece of beef is selected at random, which of the following is the best estimate of the probability that it will be a flank steak with high fat content?

- A. 0.086
- B. 0.161
- C. 0.223
- D. 0.386

25. A scatterplot was drawn to represent the relationship between two variables. If the correlation coefficient of the two variables was 0.95, which of the following could be the scatterplot?



COMPETENCY 013

5

- 26. One number is to be randomly selected from the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}. What is the probability of selecting a number that is odd or is a multiple of 3 ?
 - A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. $\frac{5}{6}$

27. Use the table below to answer the question that follows.

n	n^2	Remainder when n^2 is divided by 4
8	64	0
9	81	1
10	100	0
11	121	1

A student constructed the table shown and conjectured that the square of an integer has remainder 0 or 1 when divided by 4. Which of the following would be the best approach for a teacher to suggest to the student to prove or disprove this conjecture?

- A. Look for a counterexample using prime numbers as values for n
- B. Construct a proof by induction on the set of positive integers
- C. Examine the cases of n = 2k and n = 2k+1, where k is an integer, and compute n^2 in each case
- D. Examine the cases of n = 3k, n = 3k + 1, and n = 3k + 2, where k is an integer, and compute n^2 in each case

COMPETENCY 015

28. Use the identity below to answer the question that follows.

 $1 + 3 + 5 + \dots + (2n - 1) = n^2$ for all integers $n \ge 1$

Which of the following would be the best technique to use to prove the identity shown?

- A. Binomial expansion
- B. Mathematical induction
- C. Prime factorization
- D. Proof by contradiction

- 29. A shopper sees some gloves on sale for 20% off their original price of \$28. The next week the gloves have been reduced another 15% off the sale price. If there is a 6% sales tax, how much would the shopper now pay for the gloves?
 - A. \$16.52
 - B. \$17.11
 - C. \$19.29
 - D. \$20.18

COMPETENCY 015

30. Use the conjecture in the box below to answer the question that follows.

If the integer *n* can be factored as n = ab for positive integers *a* and *b* and if *p* is a factor of *n*, then *p* is a factor of *a* or *p* is a factor of *b*.

A student makes the conjecture shown. Which of the following is a <u>counterexample</u> that a teacher could use to show the student that the conjecture is false?

A. n = 24 = (8)(3) and p = 2

B. n = 35 = (5)(7) and p = 7

C. n = 42 = (6)(7) and p = 13

D. n = 48 = (16)(3) and p = 12

- 31. Which of the following activities would best foster sixth-grade students' development of the mathematical concept of function?
 - A. Each student uses 30 one-inch squares to make a variety of different shapes with the same area; the class then makes a table showing characteristics of each figure, and students discuss which shape has the smallest perimeter
 - B. Students measure various properties (e.g., mass, length) of a number of three-dimensional objects; they then discuss the number of quantitative descriptions that can be assigned to each object
 - C. Students measure the perimeters of six squares of different sizes and fill in a table with the headings "Length of side" and "Perimeter;" they then attempt to predict values for larger and smaller squares given the data they have obtained
 - D. Students measure the perimeter and determine the area of each of a number of regular geometric figures; they then enter the data in a table and discuss any patterns they observe

- 32. A seventh-grade mathematics teacher notices that several students are having difficulty remembering definitions for math terms discussed in class. Which of the following independent exercises would be most useful to ensure that all the students in the class understand and remember mathematical vocabulary?
 - A. Ask students to keep personal lists of definitions of mathematical terms they don't understand and periodically discuss the lists with the students
 - B. Provide all students with a list of definitions that is copied from the main text for all mathematical terms used in the class
 - C. Ask students who are familiar with the mathematical vocabulary used in the class to informally tutor those students who are having difficulty
 - D. Provide extra time at the end of each class for students to ask questions about the mathematical vocabulary used during that day's lesson

- 33. Which of the following best illustrates an application of Piaget's theory of intellectual development in a lesson on the number π for sixth-grade students at the concrete operational stage?
 - A. A teacher has students measure the circumference and diameter of a variety of circular objects; the students then compare the ratios of circumference to diameter
 - B. A teacher presents a lecture on the development and use of the number π during Babylonian and Egyptian times; the teacher concludes by discussing Archimedes's method for calculating π
 - C. A teacher defines π as the ratio of the circumference to the diameter of a circle and solves several different problems; students are then asked to solve a series of similar problems with different numbers
 - D. A teacher has students work in groups of two to solve a series of problems involving π ; the teachers tells students that a prize will be given to members of the first three groups that successfully solve all of the problems

- 34. A teacher is planning a unit on proportional relationships. Which of the following topics could the teacher include in the unit?
 - A. Properties of similar triangles
 - B. Solving quadratic equations
 - C. Finding formulas for the perimeters of figures
 - D. Solving problems with the Pythagorean theorem

- 35. Two groups of students are using a dynamic geometry software program to investigate the properties of quadrilaterals. The first group has concluded that the diagonals of a quadrilateral always bisect each other. The second group of students is not convinced. Which of the following should the teacher encourage the students to do to resolve the apparent contradiction?
 - A. Assume that the diagonals of a quadrilateral don't bisect each other and derive a contradiction
 - B. Find an example of a quadrilateral with diagonals that do not bisect each other
 - C. Formally prove that if the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram
 - D. Use informal reasoning to show that the diagonals of a quadrilateral create two triangles with an adjacent side

COMPETENCY 019

- 36. Students in a mathematics class have completed an activity in which they found the areas of rectangles by dividing the rectangles into smaller squares and adding up the areas of the smaller squares. Which of the following would be the best method for the teacher to use to assess the students' ability to apply this method of calculating area?
 - A. Calculate the area of an equilateral triangle by dividing it into smaller equilateral triangles
 - B. Calculate the area of a circle by dividing it into smaller circles
 - C. Calculate the area of a regular pentagon by dividing it into smaller squares
 - D. Calculate the area of a regular hexagon by dividing it into smaller regular hexagons

- 37. A fifth-grade teacher is beginning a new unit on geometry. The teacher would like to identify which aspects of the new topic are most difficult for students to understand in order to be able to adjust her lesson plans as the unit progresses. Which of the following assessment methods would be most appropriate for achieving this goal?
 - A. Periodic surprise quizzes
 - B. Regular peer reviews of other students' work
 - C. Performance assessments at the beginning and end of the unit
 - D. Informal observations and interviews during each class

ANSWER KEY				
Question Number	Correct Answer	Competency		
1	В	001		
2	D	001		
3	А	002		
4	В	002		
5	С	003		
6	D	003		
7	D	004		
8	А	007		
9	В	005		
10	В	005		
11	А	006		
12	С	006		
13	С	006		
14	С	008		
15	В	008		
16	В	009		
17	А	009		
18	А	010		
19	D	011		
20	D	011		
21	С	012		
22	D	012		
23	С	013		
24	А	013		
25	В	014		
26	С	013		
27	С	015		
28	В	015		
29	D	016		
30	D	015		
31	С	017		
32	А	017		
33	А	018		
34	А	018		
35	В	018		
36	А	019		
37	D	019		

Answer Key

5

Chapter 6

Are You Ready? – Last Minute Tips

$\star \star \star$

PREPARING TO TAKE THE TEST

CHECKLIST

Complete this checklist to determine if you are ready to take your test.

- \checkmark Do you know the testing requirements for your teaching field?
- \checkmark Have you followed the test registration procedures?
- ✓ Have you reviewed the test center identification document requirements in the *Registration Bulletin* or on the ETS TEXES website at **www.texes.ets.org**?
- \checkmark Do you know the test frameworks that will be covered in each of the tests you plan to take?
- ✓ Have you used the study plan sheet at the end of this booklet to identify what content you already know well and what content you will need to focus on in your studying?
- ✓ Have you reviewed any textbooks, class notes and course readings that relate to the frameworks covered?
- ✓ Do you know how long the test will take and the number of questions it contains? Have you considered how you will pace your work?
- \checkmark Are you familiar with the test directions and the types of questions for your test?
- ✓ Are you familiar with the recommended test-taking strategies and tips?
- ✓ Have you practiced by working through the sample test questions at a pace similar to that of an actual test?
- ✓ If constructed-response questions are part of your test, do you understand the scoring criteria for these questions?
- ✓ If you are repeating a test, have you analyzed your previous score report to determine areas where additional study and test preparation could be useful?

THE DAY OF THE TEST

You should have ended your review a day or two before the actual test date. Many clichés you may have heard about the day of the test are true. You should:

- Be well rested.
- Take the appropriate identification document(s) with you to the test center (identification requirements are listed in the *Registration Bulletin* and on the ETS TExES website at **www.texes.ets.org**).
- Take 3 or 4 well-sharpened soft-lead (No. 2 or HD) pencils with good erasers.
- Eat before you take the test.
- Be prepared to stand in line to check in or to wait while other test takers are being checked in.
- Stay calm. You can't control the testing situation, but you can control yourself. The test administrators are well trained and make every effort to provide uniform testing conditions, but don't let it bother you if a test doesn't start exactly on time. You will have the necessary amount of time once it does start. Using the *Reducing Test Anxiety* booklet in the days before you test may be helpful in mentally and emotionally preparing yourself to test. It is available free at **www.texes.ets.org**.

You can think of preparing for this test as training for an athletic event. Once you have trained, prepared and rested, give it everything you've got. Good luck.

Appendix A

Study Plan Sheet



A

STUDY PLAN SHEET

	STUDY PLAN					
Content covered on test	How well do I know the content?	What material do I have for studying this content?	What material do I need for studying this content?	Where can I find the materials I need?	Dates planned for study of content	Date completed

Appendix B

Preparation Resources

$\star \star \star$

PREPARATION RESOURCES

The resources listed below may help you prepare for the TExES test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

JOURNALS

Mathematics Teacher, National Council of Teachers of Mathematics.

Mathematics Teaching in the Middle School, National Council of Teachers of Mathematics.

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ONLINE RESOURCES

Internet4Classrooms — www.internet4classrooms.com

Pearson Prentice Hall - www.phschool.com

Pearson Welcome K-12 AP Teacher! — www.pearsonhighered.com/educator/K-12_AP_teacher.page

